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Building the Family Nest: Premarital Investments, Marriage Markets, and Spousal Allocations

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We develop a transferable utility model of the household in which the marriage market is characterized by (negative or positive) assortative matching, and spousal allocations are determined by premarital investments. We demonstrate that all sharing rules along the assortative order support efficient outcomes both in terms of premarital investments and intra-household allocations. The efficiency of premarital choices and household allocations then enables us to show that, for each couple, the marriage market generates a unique and maritally sustainable sharing rule that is a function of the distribution of premarital endowments and the sex ratios in the market. According to our results, transfers among spouses occur on two margins: premarital investments and intra-marital spousal allocations. Asymmetries in the sex ratios in the marriage markets produce gender differences in premarital investments and consumption that are larger for individuals with small premarital endowments than those with larger endowments. A corollary of these findings is that, when men are in short supply in the marriage markets, women can invest more than men even when the returns to investment are lower or the costs are higher for women.

1. INTRODUCTION

Recent literature has shown that a treatment of the household as a single decision-making unit is not consistent with a growing body of empirical evidence on intra-household allocations.[1] Instead, the "collective" view, in which intra-household allocations are assumed to be efficient and individual members of the family are treated as the core decision-makers, has emerged as a compelling alternative.[2]

The collective household models suggest—and the empirical evidence supports—the notion that relative spousal incomes influence household allocations.[3] While the collective approach to household behaviour takes spousal incomes as given, these incomes are determined at least in part by decisions individuals make prior to marriage.[4] Thus, implicit in the construction of

1. See, for example, Browning, Bourguignon, Chiappori and Lechene (1994), Udry (1996), and Chiappori, Fortin and Lacroix (2002).

2. The generalized version of this literature was spearheaded by Becker (1981) and developed further by Chiappori (1988, 1992).

3. See, for instance, Thomas (1990) and Browning *et al.* (1994).

4. In models of the household where spousal incomes are pure public goods, such decisions can lead to inefficient premarital choices and intra-household allocations, although the efficiency of premarital investments can be restored as a result of spousal competition in the markets for marriage. For instance, Bergstrom, Blume and Varian (1986) demonstrate that economic agents under-invest prior to the beginning of a cooperative stage during which their potential partners'

the collective framework is the idea that premarital investments influence wage earnings and, hence, the intra-marital sharing rules. Nonetheless, due to the lack of an integrated theoretical framework, the empirical studies in this area remain agnostic with respect to the factors that could determine intra-marital allocations.

In this paper, we incorporate premarital investments and spousal matching into a "collective" household model where marital matching is assortative, and an endogenous sharing rule forms the basis of intra-household allocations. Extending the collective household framework in such fashion enables us to make important contributions to the existing literatures on one-to-one matching models, economics of the family, and development. First, we are able to identify how the intra-marital sharing rules that emerge endogenously in the markets for marriage affect individuals' investment choices prior to household formation. As we shall detail below, for instance, the demand for and the supply of spouses in the marriage markets as well as the gender differences in the costs of and returns to premarital investments need to be accounted for to explain patterns of spousal education, earnings, and consumption.

Second, and more generally, our model extends the one-to-one matching models with pure exchange in large economies to incorporate a production/investment stage prior to the process of matching.[5] By doing so, it demonstrates that matching in large marriage markets supports the Walrasian equilibrium despite the fact that production and investment decisions prior to matching yield externalities.

Third, because we show that both premarital investments and household allocations are made efficiently (in marriage markets without frictions), we are able to identify the theoretical determinants of the intra-household sharing rules.[6] Our main finding is that in many cases a unique sharing rule emerges as a maritally sustainable outcome for each couple. The incorporation of premarital investments into a model of intra-household allocations highlights that the sharing rules in general and spousal prices in particular reflect adjustments both in premarital choices and intra-marital spousal allocations. For instance, when the marriage market becomes short on men, not only should their premarital investments decrease, but also their intra-marital consumption relative to their wives should increase. In general, both the distribution of spousal premarital endowments and the sex ratios in the marriage markets help determine the sharing rules. However, the role of the sex ratios in determining intra-household allocations increases as the rank of couples in the assortative order rises. That is, for high-ranking couples, the intra-household allocations are influenced more by the imbalances in the sex ratios, whereas those for lower-ranking couples are influenced more by relative spousal endowments.

Fourth, endogenizing the intra-marital sharing rules in the presence of premarital investments generates some novel empirical predictions as well as explanations for some stylized facts. For example, the equilibrium sharing rules we derive below are such that asymmetries in the sex ratios in the marriage markets produce gender differences in premarital investments and consumption that are larger for individuals with small premarital endowments than those with large endowments. This is because individuals on the long side of the market and in lower-ranked marriages compete more intensely for their scarce mates (than richer individuals on the same side of the market) and, accordingly, surrender more of their marital surplus. An important corollary of these findings is that, when men are in short supply in the marriage markets, women can invest

incomes are pure public goods. Recently, Peters and Siow (2002) have combined such a model with assortative spousal matching to reveal how competition in large marriage markets restores Pareto efficiency.

5. For models of pure exchange with one-to-one matching, see Gretsky, Ostroy and Zame (1992, 1999) and Ostroy and Zame (1994).

6. Hereafter, we refer to equilibria outcomes as *unconditionally Pareto efficient* if neither premarital investments nor intra-marital allocations can be altered to make one spouse strictly better off while leaving the other no worse off. We define outcomes as *conditionally Pareto efficient* if, given the choices spouses have made prior to marriage, intra-marital allocations cannot be altered to make one spouse better off while not affecting the other spouse.

more than men premaritally even when the returns to their investments are lower or the costs are higher for women.

It is well documented that while birth rates of males are slightly higher than that of females in the U.S., the mortality rate for men rises much faster with age. In addition, there is some degree of positive assortative matching among spouses by age.[7] Hence, in most marriage markets in the U.S. except those for the very young, men are in short supply. Additionally, there has been a rapid increase in the education of women relative to men in the U.S. despite the fact that the gender wage gap has narrowed, but still exists. Moreover, the proportions of women with some years of college, a college degree, and advanced degrees have exceeded those for men in the U.S. since 2003. For some time, this has been the trend in other industrialized countries as well.[8] Our model reconciles these facts with an emphasis on marital matching.[9]

In developing countries, premarital investments often take the form of dowries and bride prices. According to our model, changes in the sex ratios influence premarital investments, however broadly or narrowly the latter may be defined. Indeed, as Anderson (2003) points out, the persistent increase in dowries in India in the last five decades can be attributed to changes in the sex ratios in the marriage markets for low- and high-caste single individuals. In Anderson's work, the main focus is on the impact of a wider wealth distribution on marriage market clearing and dowries, whereas we establish how changes in the supply and demand for spouses could influence the allocations of resources both in preparation for and during the marriage. Hence, taking the recent literature on intra-household allocations seriously, our model shows that increases in dowries ought to be accompanied by lower intra-marital allocations for wives of lower marriage ranks.

Our model also produces some stark predictions regarding how educational attainment among married individuals relative to that among singles would evolve with changes in the gender gap in the returns to and costs of education.[10] Depending on whether spousal attributes are complement or perfect substitutes in marital production, we document that changes in the gender wage gap or differences in the cost of education would have a differential effect on premarital investments. In particular, a narrower gender gap would induce women to invest relatively more regardless of the marital production technology. However, if spousal attributes are complements, and there are more women than men, women in the lower ranks would invest relatively more than those in higher ranks. Moreover, regardless of their gender, singles would invest less than similarly endowed married individuals. But if spousal attributes are substitutes, and there are more women than men in the marriage markets, single women, who would be those at the top of the female endowment distribution, would be more educated than their married counterparts.

2. RELATED LITERATURE

This paper sits at the juncture of three strands in the economics literature. The first strand is on "collective" household models and early- and late-generation marital bargaining models. These allow for differences between spouses to affect the choices households make by relying on a sharing rule or an intra-household bargaining mechanism. The common analytical basis of this strand

7. Choo and Siow (2005).

8. See Goldin, Katz and Kuziemko (2006) for evidence from the U.S. and Browning, Chiappori and Weiss (in progress) for data from other industrialized countries.

9. An alternative explanation for this trend is provided by Chiappori, Iyigun and Weiss (2006): they show that combining spousal matching with labour market discrimination that is weaker (stronger) for more (less) educated women can induce women to invest relatively more than men (even when the sex ratio is in balance).

10. By definition, a gender gap in the cost of education refers to the gender differences in the cost of educational attainment. While these costs may be explicit, in practice they usually take the form of cultural attitudes or social norms that create impediments for women and make it relatively more costly for them to obtain education.

is that family members with potentially different preferences make Pareto-efficient household decisions. Among the earliest examples of the collective models are Becker (1981), Chiappori (1988, 1992), and Bourguignon and Chiappori (1994) and those of exogenous marital bargaining are Manser and Brown (1980), McElroy and Horney (1981), and Sen (1983). Each of these models assumes that the sharing rule or the bargaining power of the two sexes is determined exogenously and that couples have different preferences over the choice sets. In two exceptions, Basu (2006) and Iyigun and Walsh (2006) suggest models that treat the bargaining power of the sexes as determined endogenously according to actual relative earnings. Neither of these models examines how the existence of premarital investments impacts intra-marital allocations in a collective household setting.

The second strand of the literature to which this work is related includes papers that explore how matching influences premarital investments in models where spousal incomes are treated as marital public goods. Earlier works in this line, such as Bergstrom *et al.* (1986) and MacLeod and Malcomson (1993), have shown that the equilibrium educational investments are below the Pareto-efficient level when premarital investments are a public good in marriage. These papers do not take into account how endogenous matching might influence premarital investments. Once marital matching is endogenized, Peters and Siow (2002) demonstrate that families make investments in education that are Pareto optimal. According to their results, the combination of large marriage markets, assortative matching, and bilateral efficiency guarantees that the equilibrium distribution of premarital investments is efficient. This is due to the fact that, when spousal wealth is a public good in marriage, the competitive marriage market and the assortative matching that occurs within it guide families to indirectly and reciprocally compensate each other for the investments that they make in their own children. No paper in this strand addresses how premarital investments might be influenced in a collective household setting.

The third strand to which this paper is related combines intra-household allocations with marital matching. Here, our work is most similar to Becker and Murphy (2000) and Browning, Chiappori and Weiss (2003). As far as we know, these two papers and ours represent inaugural attempts to broaden the collective approach to cover aspects of household formation that precede marriage.[11] Becker–Murphy and Browning–Chiappori–Weiss share similarities in that they both merge the collective household model with marital sorting to explore the implications of spousal matching. In both works, however, the resources each spouse contributes to the marriage are taken as given. In contrast, we extend the collective household to include premarital investments in order to examine, among other things, the efficiency implications of premarital investments and marital sorting for the collective model.

Before we begin, we should also make explicit two features of our model: one, it is based on transferable utility and marriage markets that operate frictionlessly. As such, the couples' outside prospects—be it their utility as singles or in another potential marriage—play an independent role in whether household allocations are made efficiently. These aspects of our model stand in contrast to those of Siow and Peters, where utility is not transferable and commitment is not required. However, neither the lack of a commitment mechanism nor the existence of marriage-specific investments, which might lower spouses' outside options influences our main findings. In the analysis below, the ability to remarry acts as a substitute for commitment, and the existence of singles who are potential mates helps spouses who specialize in home work and make potentially costly marriage-specific investments capture all of the marital surplus in an existing marriage.

11. It is important to note that, while theoretical papers have just begun to incorporate premarital investments and spousal matching into models of intra-household allocations, empirical papers, which recognize that spousal incomes are endogenous, have existed for a while (see, for instance, Lundberg, Pollak and Wales, 1997; Qian, 2005). Also, recent structural empirical work has focused on incorporating spousal matching into models of household decision-making (examples include Del Boca and Flinn, 2005; Choo, Seitz and Siow, 2006).

This result holds even when there are some search frictions in spousal matching. While we model spousal matching in the marriage markets as a frictionless process, the endogenously sustained efficiency result depends on the existence of singles in equilibrium. As such, the main conclusions of the model below would remain intact in models with spousal search frictions.[12]

Two, our work borrows from and complements the findings of Gretsky *et al.* (1992) who demonstrate that (i) solutions to one-to-one matching assignments exist when matching attributes on both sides are distributed over continuous supports and that (ii) these solutions correspond to the Walrasian equilibria of the associated market economy. We extend the findings of Gretsky *et al.* by adding a stage of prematching investments, which is akin to incorporating production into an exchange economy. We borrow from their work because the proof of the existence of equilibria in our model is a simple extension of the Grestky–Ostroy–Zame existence result.

The remainder of our paper is organized as follows: in Section 3, we describe spousal preferences, endowments, and the marital production technology. In Section 4, we examine the marital outcomes and their stability conditional on a given set of premarital decisions. In Section 5, we discuss individuals' premarital investments and how the matching process in the marriage markets and the subsequent determination of intra-household allocations influence premarital investments. In Section 6, we establish the properties of the unconditionally Pareto-efficient frontier. In Section 7, we extend our analysis to cover cases in which spousal inputs in marital production are perfect substitutes. In Section 8, we present some examples to further illustrate the properties of our model. In Section 9, we conclude.

3. THE BASIC MODEL

The economy is made up of individuals who live for two periods. The total mass of women in the economy is equal to F and that of men is equal to M. Let $G(N)$ and $H(N)$, respectively, be measures of the sets of men and women whose endowments lie in the set N and let r, $r \lesseqgtr 1$, denote the measure of men relative to women.[13] All individuals are endowed with an initial wealth of y, where $y \in [y^{\min}, y^{\max}]$, $y^{\max} > y^{\min} \geq 0$.

When young, individuals allocate their wealth to consumption and a form of investment that augments their future incomes.[14] When they get old, individuals either marry or stay single but they all work and consume. The efficiency units of labour for each individual is linearly proportional to his or her premarital investment, ω_i.

Individual preferences are defined over first- and second-period consumptions, c_i^1 and c_i^2. We assume that preferences of men and women are represented by the following inter-temporal utility function, respectively:

$$U = u_1(c_m^1) + u_2(c_m^2) \tag{1}$$

and

$$V = \upsilon_1(c_f^1) + \upsilon_2(c_f^2), \tag{2}$$

12. In models with search frictions, such as Hadfield (1999), Chiappori and Weiss (2000, 2004), and Baker and Jacobsen (2003), there exist frictions in the marital matching process (as a result of which not all meetings lead to marriages). While both models with and without frictions have their own merits, the former in general sustain inefficient choices. A similar result would attain in a modified version of the model below as well, because in choosing their optimal premarital investments, individuals would account for the fact that they might remain single in adulthood. As a result, conditional on the state of being married, couples would invest below the efficient levels. That noted, *ex ante* efficiency would still be attained since individuals' choices when young would accurately reflect the uncertainty regarding marriage in the future. This would again make it possible to pin down the determinants of intra-household sharing rules.

13. Hence, $r = 1$ if there are equal measures of men and women, $r < 1$ if there are more women than men, and $r > 1$ if there are more men than women.

14. For instance, investment in formal and/or informal education. But an alternative interpretation of these premarital investments is consistent with dowries or bride prices that spouses contribute to marriage, which in turn influence each partner's allocations in marriage.

where the functions U and V satisfy, $\forall\ c_i^1, c_i^2 \in [0, y_i]$, $i = f, m$, $u_1', u_2', \upsilon_1', \upsilon_2' > 0$, $u_1'', u_2'', \upsilon_1''$, $\upsilon_2'' < 0$, and $u_1(\cdot)$, $\upsilon_1(\cdot)$ are invertible.

The young invest ω_i, $\omega_i \leq y_i$, $i = f, m$, in order to augment their future incomes.[15] When they get older, individuals can either remain single or match in the marriage market according to their premarital investments ω_i. First-period investments translate into second-period consumptions via the production function $h(\omega_m^*, \omega_f^*)$.

If partners share a public good in marriage, their spousal incomes are complements in marital production so that $h_{\omega_m^*\omega_f^*}(\omega_m^*, \omega_f^*) > 0$. In Sections 3–6, we assume that the second-period production function $h(\omega_m^*, \omega_f^*)$ is supermodular so that $h_{\omega_m^*\omega_f^*}(\omega_m^*, \omega_f^*) > 0$. Therefore, the marital production function explicitly incorporates gains from marriage and generates positive assortative matching in the marriage market equilibrium.

Definition 1. For a couple with the premarital investment levels ω_m^* and ω_f^*, $h = h(\omega_m^*, \omega_f^*)$ defines the marital production function, where $h = h(\omega_m^*, \omega_f^*)$ is twice differentiable and supermodular. Hence, $\forall \omega_m^*, \omega_f^* > 0$, $h_{\omega_m^*}(\omega_m^*, \omega_f^*)$, $h_{\omega_f^*}(\omega_m^*, \omega_f^*) > 0$, $h_{\omega_m^*\omega_m^*}(\omega_m^*, \omega_f^*)$, $h_{\omega_f^*\omega_f^*}(\omega_m^*, \omega_f^*) < 0$, and $h_{\omega_m^*\omega_f^*}(\omega_m^*, \omega_f^*) > 0$.

Lemma 1. *Given Definition* 1, *positive assortative matching by spousal investment levels, ω_m^* and ω_f^*, is optimal in the marriage market.*

Proof. See Appendix. ||

Let ω_i^s, $i = f, m$, denote the optimal investment level of an individual who remains single. The second-period consumption of a single male is given by $h(\omega_m^s, 0)$ and that of a single female is given by $h(0, \omega_f^s)$.[16] Thus, for single men and women, utility in the second period of life equals

$$u_2[h(\omega_m^s, 0)] \quad \text{and} \quad \upsilon_2[h(0, \omega_f^s)], \tag{3}$$

respectively. For single individuals, optimal levels of premarital investment, ω_i^s, $i = f, m$, are

$$\omega_i^s = \begin{cases} \arg\max U = u_1(y_m - \omega_m^s) + u_2[h(\omega_m^s, 0)] & \text{if } i = m, \\ \arg\max V = \upsilon_1(y_f - \omega_f^s) + \upsilon_2[h(0, \omega_f^s)] & \text{if } i = f. \end{cases} \tag{4}$$

Lemma 2. *The optimal investment levels of single men and women, respectively, satisfy*

$$\begin{aligned} u_1'(y_m - \omega_m^s) &= u_2'[h(\omega_m^s, 0)]h_{\omega_m}(\omega_m^s, 0), \\ \upsilon_1'(y_f - \omega_f^s) &= \upsilon_2'[h(0, \omega_f^s)]h_{\omega_f}(0, \omega_f^s)\,. \end{aligned} \tag{5}$$

Proof. Follows immediately from Definition 1 and equations (3) and (4). ||

4. STABLE MARITAL MATCHINGS AND CONDITIONAL EFFICIENCY

We now turn to the outcomes in the marriage market, taking first-period investment decisions as given. Let the pair (ω_m^*, ω_f^*) represent a given set of premarital investments made by a male and a

15. For simplicity, we assume that there are only pecuniary costs of education, although extending the model to allow time costs would not alter our main results. See footnote 20.

16. The supermodularity of the marital production function implies that $\forall \omega_i > 0$, $i = f, m$, $h(0, \omega_f)$, $h(\omega_m, 0) < h(\omega_m, \omega_f)$, $h_{\omega_f}(0, \omega_f) < h_{\omega_f}(\omega_m, \omega_f)$, and $h_{\omega_m}(\omega_m, 0) < h_{\omega_m}(\omega_m, \omega_f)$.

female, where $\omega_i^* \in [\Omega_i^{\min}, \Omega_i^{\max}]$, $i = f, m$.[17] Let $\hat{G}(N)$ and $\hat{H}(N)$, respectively, be measures of the sets of males and females whose premarital investments lie in the set N. Conditional on the set of premarital investments, (ω_m^*, ω_f^*), the sharing rule that divides second-period consumption between the couple and supports a stable assignment in the marriage market must be such that the utilities achieved by the partners satisfy

$$c_m^2(\omega_m^*) + c_f^2(\omega_f^*) \geq h(\omega_m^*, \omega_f^*) + g, \tag{6}$$

where g, $g \geqslant 0$, represents a common gain from marriage (that is measured in consumption terms and is unrelated to spousal inputs) and where $c_i^2(\omega_i^*)$, $i = f, m$, denotes the second-period consumption offered to an individual with investment ω_i^* by a potential partner with ω_{-i}^*.

Note that, if equation (6) holds as a strict inequality for any given pair (ω_m^*, ω_f^*), then the consumption demands of the couple exceed their marital output and the matching cannot be sustained. However, if (6) holds as an equality for a match (ω_m^*, ω_f^*), that pairing would be maritally feasible. In addition, recall that due to the supermodularity of the marital output function, $\forall(\omega_m^*, \omega_f^*) \gg 0$, $h(0, \omega_f^*) + h(\omega_m^*, 0) < h(\omega_m^*, \omega_f^*)$. Thus, the function $h(\omega_m, \omega_f)$ explicitly incorporates "gains" from marriage, which are allocated between the two spouses.

Definition 2. Conditional on a pair of premarital investment levels, (ω_m^*, ω_f^*), the allocations in marriage, $c_m^2(\omega_m^*) c_f^2(\omega_f^*)$, define a marriage market equilibrium if for all pairs (ω_m^*, ω_f^*) in the set of married couples

1. $r[1 - \hat{G}(\omega_m^*)] = 1 - \hat{H}(\omega_f^*)$;
2. $\forall \omega_m^* \in [\Omega_m^{\min}, \Omega_m^{\max}]$, $c_m^2(\omega_m^*) = g + \arg\max_{\omega_f^*}[h(\omega_m^*, \omega_f^*) - c_f^2(\omega_f^*)]$;
3. $\forall \omega_f^* \in [\Omega_f^{\min}, \Omega_f^{\max}]$, $c_f^2(\omega_f^*) = g + \arg\max_{\omega_m^*}[h(\omega_m^*, \omega_f^*) - c_m^2(\omega_m^*)]$.

Part 1 of the definition is the marriage market clearing condition. It guarantees that, by assortative matching, each husband who invests ω_m^* or more will be able to match with a wife who invests ω_f^*. Thus, we have the following spousal matching functions:

$$\omega_m^* = \Phi\left\{1 - \frac{1}{r}[1 - \hat{H}(\omega_f^*)]\right\} \equiv \phi(\omega_f^*) \tag{7}$$

and

$$\omega_f^* = \Psi\left\{1 - r[1 - \hat{G}(\omega_m^*)]\right\} \equiv \psi(\omega_m^*), \tag{8}$$

where $\Phi \equiv \hat{G}^{-1}$ and $\Psi \equiv \hat{H}^{-1}$. Note that either of the functions $\phi(\omega_f^*)$ and $\psi(\omega_m^*)$ fully describe the nature of spousal matching.

Parts 2 and 3 of our definition indicate that all individuals choose their spouses optimally in order to maximize their gains from marriage and as implied by equation (6). Accordingly, these two conditions yield the following two first-order conditions:[18]

$$c_m^2(\omega_m^*)' = h_{\omega_m^*}[\omega_m^*, \psi(\omega_m^*)] \tag{9}$$

and

$$c_f^2(\omega_f^*)' = h_{\omega_f^*}[\phi(\omega_f^*), \omega_f^*]. \tag{10}$$

17. Note that the variables $\Omega_i^{\min}$ and $\Omega_i^{\max}$, which represent the endogenously determined upper and lower bounds of the support of ω_i across all individuals (married or single), increase with the lower and upper bound of the support of the endowments, which equal $y^{\min}$ and $y^{\max}$.

18. Note that we express the first-order conditions after applying the envelope theorem.

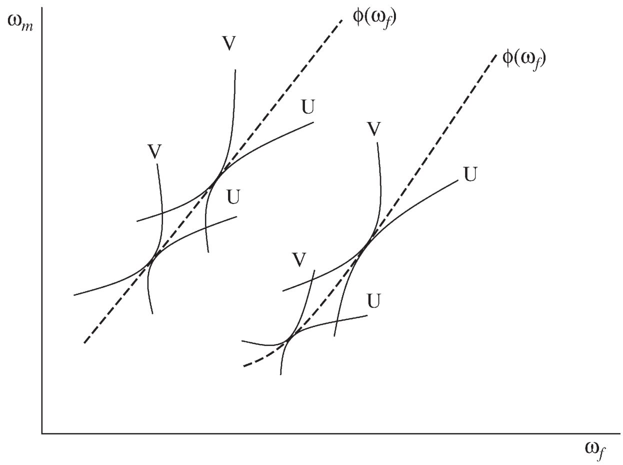

FIGURE 1
The marital matching function

Equations (9) and (10) characterize the sharing rules that hold in equilibrium. The key point is that the second-period consumption of each spouse in marriage changes according to not who (s)he marries but based on the marginal contribution the spouse makes to marital output. Note that (9) and (10) hold for all married couples along the assortative order.

In Figure 1, we depict two possible equilibria that could emerge in the marriage market conditional on the investments chosen in the first period. The premarital investment levels of the women are shown on the horizontal axis and those of the men are on the vertical axis. The two upward-sloping dashed lines represent two different equilibrium matching functions $\phi(\omega_f^*)$. The upward convex curves are the indifference curves of the wives and those that are convex downward are the indifference curves of the husbands. They both incorporate the sharing rules associated with each potential spousal match. Due to the assortative matching equilibrium, couples for whom the husband has higher initial endowment, y_m, invest more than those for whom the husband has a lower initial endowment. If distributional factors favour men more than they do women, then the equilibrium matching function will tend to shift to the right leading to more investment by the wives and less by the husbands.

To derive the intra-marital allocations of each spouse along the assortative marital order, we integrate the expressions in (9) and (10):

$$c_m^2(\omega_m^*) = k + \int_{\omega_m^0}^{\omega_m^*} h_{\omega_m^*}[s, \psi(s)]ds \tag{11}$$

and

$$c_f^2(\omega_f^*) = k' + \int_{\omega_f^0}^{\omega_f^*} h_{\omega_f^*}[\phi(t), t]dt, \tag{12}$$

where $k + k' = g + h(\omega_m^0, \omega_f^0)$, and ω_i^0 represents the investment level of the lowest ranked individual of type i, $i = f, m$, who marries. Note that, for a couple in the lowest assortative order, it has to be the case that $k = h(\omega_m^s, 0)$ if $r > 1$ and $k' = h(0, \omega_f^s)$ if $r < 1$. Finally, if

$r = 1$, multiple equilibria are possible, and all we can say is that $k + k' = h(\Omega_m^{\min}, \Omega_f^{\max}) + g$.[19] Essentially, these allocations ensure that, when the sex ratio is not balanced (*i.e.* $r \neq 1$), spouses in the lowest assortative order invest as if they were single, and the spouse from the overabundant group receives his or her reservation level of utility, which is given by equation (4).

In our first proposition, we note that stable marriage market equilibria exist conditional on the distribution of premarital investments. We also establish the conditions under which the process of spousal matching generates a unique equilibrium:

Proposition 1. *Given Definitions* 1, 2, *and conditional on the distribution of premarital investments,* $\hat{G}(\omega_m^*)$ *and* $\hat{H}(\omega_f^*)$*, stable spousal matching equilibria exist. When* $r \neq 1$*, the equilibrium is unique.*

Proof. See Appendix. ‖

5. PREMARITAL INVESTMENTS

We now consider marriage market outcomes with endogenous investments. In equilibrium, males and females choose their levels of premarital investments according to

$$\max_{\omega_m} U = u_1(y_m - \omega_m) + u_2[c_m^2(\omega_m)], \tag{13}$$

subject to equation (11), and

$$\max_{\omega_f} V = \upsilon_1(y_f - \omega_f) + \upsilon_2[c_f^2(\omega_f)], \tag{14}$$

subject to equation (12).

The solution to these problems yield the following two first-order conditions, respectively:

$$u_1'(y_m - \omega_m^*) = u_2'[c_m^2(\omega_m^*)]h_{\omega_m^*}[\omega_m^*, \psi(\omega_m^*)] \tag{15}$$

and

$$\upsilon_1'(y_f - \omega_f^*) = \upsilon_2'[c_f^2(\omega_f^*)]h_{\omega_f^*}[\phi(\omega_f^*), \omega_f^*]. \tag{16}$$

The two first-order conditions associated with the optimal premarital investments, equations (15) and (16), also implicitly yield two response functions. Let $\omega_m = f(\omega_f; y_m)$ and $\omega_f = g(\omega_m; y_f)$, respectively, represent the implicit association between a male and his potential spouse and between a female and her potential spouse as they are defined by (15) and (16). For a stable marriage market equilibrium, the functions $f(\omega_f; y_m)$ and $g(\omega_m; y_f)$ would be related to the matching functions defined by $\phi(\omega_f)$ and $\psi(\omega_m)$ because, in equilibrium, $\omega_m^* = \phi(\omega_f^*) = f(\omega_f^*; y_m)$, and $\omega_f^* = \psi(\omega_m^*) = g(\omega_m^*; y_f)$. Put differently, the marital matching functions $\phi(\omega_f)$ and $\psi(\omega_m)$ are such that $\forall(\omega_m^*, \omega_f^*)$ $\omega_m^* = \phi(\omega_f^*) = f(\omega_f^*; y_m)$ and $\omega_f^* = \psi(\omega_m^*) = g(\omega_m^*; y_f)$.

For the marriage market outcomes to be stable, the chosen premarital investment levels must satisfy an *inter-temporal stability condition*. That is, it has to be the case that the premarital investment levels of married men and women are greater than or equal to those of single men and women with identical levels of endowment, $\omega_i^* \geq \omega_i^s, i = f, m$. Otherwise, the marital matchings in the lowest assortative order cannot be maintained as the scarce-gender spouses of the lowest assortative rank would prefer to divorce their spouses and marry excess-gender single individuals

19. Note, however, that when $r = 1$ and $g = 0$, if the lower bound on investment for each sex is 0, then there is a unique equilibrium where $k = k' = 0$.

with higher levels of investment. The supermodularity of the marital output function (which implies that $h_{\omega_m^*\omega_f^*}(\omega_m, \omega_f) > 0$) ensures that the inter-temporal stability condition is met so that $\omega_i^* \geq \omega_i^s, i = f, m$.

Implicit in our model and the derivations of the first-order conditions in (15) and (16) is the assumption that individuals have no access to capital markets (and therefore that premarital investments are self-financed). The alternative is to assume full access to capital markets in human capital investments.[20] As we elaborate in footnote 20, our main results would go through under this alternative formulation, although equations (15) and (16) would have to be modified so as to equate the marginal return of premarital spousal investments to the interest rate paid on educational loans. Then, spousal material endowments would play no role in optimal education decisions but ability endowments, which would determine the rates of return from education, would.

Extending Definition 2, we formally define the *marriage market equilibrium* when individuals choose their premarital investments optimally.

Definition 3. The investments $\omega_m^*(y_m)$ and $\omega_f^*(y_f)$ and allocations in marriage, $c_m^2[\omega_m^*(y_m)]$ and $c_f^2[\omega_f^*(y_f)]$, define a marriage market equilibrium if there exist endowments, y_m and y_f, for individuals in F and M, respectively, such that

1. $r[1-\hat{G}(\omega_m^*(y_m))] = 1-\hat{H}(\omega_f^*(y_f))$;
2. $\forall \omega_m^*(y_m) \in [\Omega_m^{\min}, \Omega_m^{\max}], c_m^2(\omega_m^*(y_m)) = g + \arg\max_{\omega_f^*}[h(\omega_m^*(y_m), \omega_f^*(y_f)) - c_f^2(\omega_f^*(y_f))]$;
3. $\forall \omega_f^*(y_f) \in [\Omega_f^{\min}, \Omega_f^{\max}], c_f^2(\omega_f^*(y_f)) = g + \arg\max_{\omega_m^*}[h(\omega_m^*(y_m), \omega_f^*(y_f)) - c_m^2(\omega_m^*(y_m))]$;
4. Investments and consumption decisions satisfy the first-order conditions of equations (15) and (16).

In our second proposition, we extend Proposition 1 to account for premarital investments and the endogeneity of marital inputs:

Proposition 2. *Given Definitions* 1 *and* 3 *and the problems specified in* (13) *and* (14), *stable spousal matching equilibria exist if there exists an investment matching function* $\psi(\omega_m)$, *and*

20. See Becker (1993, pp. 92–94) for a discussion of the importance and relevance of credit market imperfections in human capital acquisition.

Although we do not develop a full-blown version of the model in this paper, consider the following modifications: suppose that $y_i, i = f, m$, denotes the innate ability of individual i, and the wage income in the second period equals $\omega_i = y_i(1+e_i{}^{\eta}), i = f, m$, with $0 < \eta < 1$, where e_i represents the education level chosen by person i in the first period before getting married. By construction, when individuals are getting their education in the first period, their education level equals 0 and their labour income equals y_i. If there are no borrowing constraints in human capital accumulation and the appropriate gross rate of interest for education finance equals R, then according to equations (1) and (2), the optimal education levels of singles would be the solutions to

$$\max_{e_m} U = u_1(y_m) + u_2[h(y_m(1+e_m^{\eta}), 0) - Re_m]$$

and

$$\max_{e_f} V = v_1(y_f) + v_2[h(0, y_f(1+e_f^{\eta})) - Re_f].$$

Thus, the optimal and efficient education levels of single men and women would, respectively, satisfy $h_1(y_m(1+e_m^*)^{\eta}, 0)y_m/e_m^{1-\eta} = R$ and $h_2(0, y_f(1+e_f^*)^{\eta})y_f/e_f^{1-\eta} = R$.

If $h_1 + h_{11}\omega_i > 0$, $\forall y_i \in [y^{\min}, y^{\max}], \omega_i \in [\Omega_i^{\min}, \Omega_i^{\max}], i = f, m$, then it follows that higher endowment individuals will acquire more education. The reader can then verify that the main results we derived with our baseline model applies in this case too.

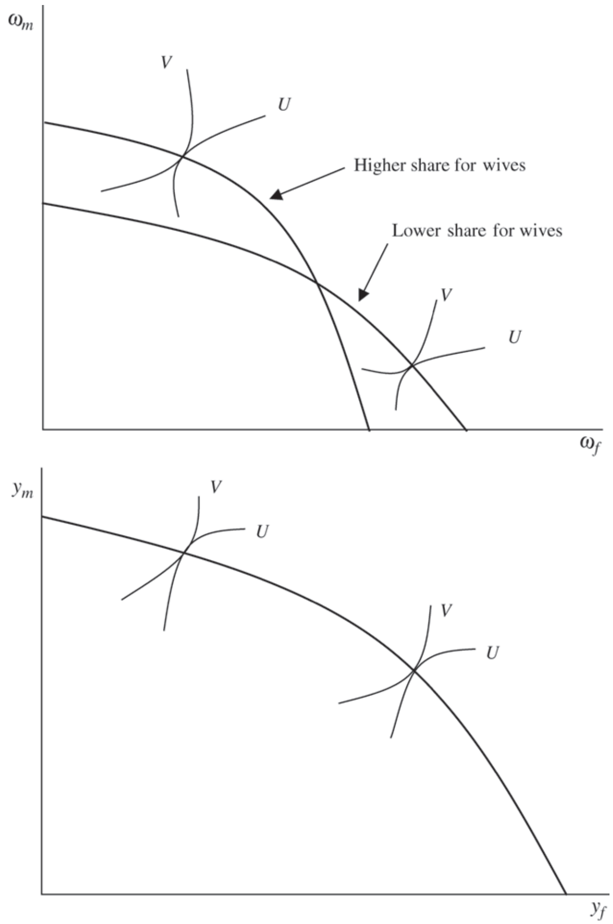

FIGURE 2
The marital contract curve

implied endogenous premarital investments $\omega_m^*(y_m)$ *and* $\omega_f^*(y_f)$*, that satisfy the relationship*

$$\gamma\left\{\omega_m^* + (u_1')^{-1}\left[u_2'\left(k + \int_{\omega_m^0}^{\omega_m^*} h_{\omega_m^*}[s,\psi(s)]ds\right) h_{\omega_m^*}[\omega_m^*,\psi(\omega_m^*)]\right]\right\}$$
$$= \psi(\omega_m^*) + (v_1')^{-1}\left[v_2'\left(h[\omega_m^*,\psi(\omega_m^*)] - k - \int_{\omega_m^0}^{\omega_m^*} h_{\omega_m^*}[s,\psi(s)]ds\right) h_{\omega_v^*}[\omega_m^*,\psi(\omega_m^*)]\right]. \quad (17)$$

When $r \neq 1$*, the equilibrium is unique.*

Proof. See Appendix. ||

While it is difficult to clearly identify regularity assumptions under which this condition for the existence of an equilibrium will be met universally, it is quite straightforward to demonstrate existence for specific examples. For instance, consider

Lemma 3. *Given Proposition* 2, *stable spousal matching equilibria exist when* $U = u_1(c_m^1) + u_2(c_m^2) = \ln(c_m^2) + \ln(c_m^2)$ *and* $V = \upsilon_1(c_f^1) + \upsilon_2(c_f^2) = \ln(c_f^2) + \ln(c_f^2)$.

Proof. See Appendix. ||

In the two panels of Figure 2, we depict the effects of more favourable marital matching for women on equilibrium outcomes (*i.e.* an increase in the sex ratio r). In the top panel, we depict two equilibria in terms of outcomes in premarital investments and for a given spousal match with the endowments of (y_m^*, y_f^*). (Note that the premarital investment levels in this panel are bounded from above at $y^{\max}$ for both the husbands and the wives.) In the bottom panel, we illustrate equilibria outcomes in terms of the endowments of matched couples. As shown in the top panel, when marriage market conditions become more favourable to women so that the spousal matching function, $\phi[\omega_f(y_f)]$, shifts to the left, all wives' overall utility level increases at the expense of their husbands. And since the adjustments in utility take place both in terms of premarital investments and intra-marital consumption, a more favourable match for the wives raises their intra-marital allocations, lowers their premarital investments, and rotates the equilibrium combinations of premarital investments clockwise. Moreover, as depicted in the lower panel, shifts in marital matching in favour of women due to an increase in r produce higher endowment grooms for all wives. As in the case of conventional collective models without premarital investments, such changes benefit all wives' intra-marital allocations holding constant the premarital levels of investment.

6. PARETO-EFFICIENT PREMARITAL INVESTMENTS AND INTRA-HOUSEHOLD ALLOCATIONS

For a given couple, the set of unconditionally efficient premarital investments and intra-household allocations can be determined by solving the following maximization problem:

$$\max_{\{\omega_f, \omega_m, c_f^2, c_m^2\}} U = u_1(y_m - \omega_m) + u_2(c_m^2) \tag{18}$$

subject to

$$V = \upsilon_1(y_f - \omega_f) + \upsilon_2(c_f^2) \geq \bar{U}_f\ , \tag{19}$$

$$c_m^2 + c_f^2 \leq h(\omega_m, \omega_f) + g, \tag{20}$$

and

$$\omega_m \leq y_m \text{ and } \omega_f \leq y_f. \tag{21}$$

In addition to the constraints of equations (19), (20), and (21), the first-order conditions for this problem are

$$u_1'(y_m - \omega_m^*) = u_2'(c_m^2) h_{\omega_m^*}(\omega_m^*, \omega_f^*) \tag{22}$$

and

$$\upsilon_1'(y_f - \omega_f^*) = \upsilon_2'(c_f^2) h_{\omega_f^*}(\omega_m^*, \omega_f^*). \tag{23}$$

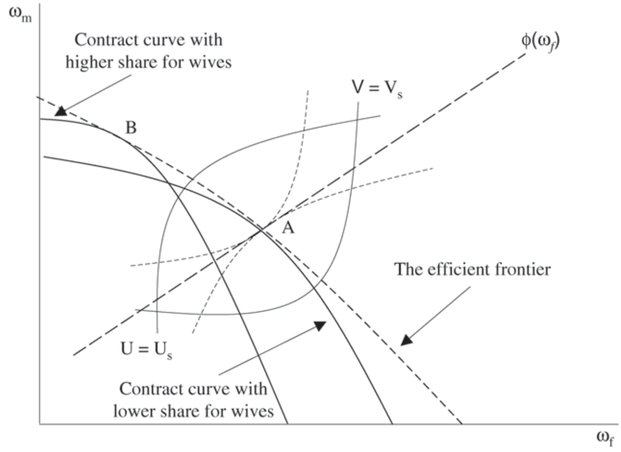

FIGURE 3

The marital contract curve and the efficient frontier

These conditions can be rewritten as

$$\frac{u_1'(y_m-\omega_m^*)-u_2'[c_m^2(\omega_m^*)]h_{\omega_m^*}[\omega_m^*,\ \psi(\omega_m^*)]}{u_2'[c_m^2(\omega_m^*)]\{c_m^2(\omega_m^*)'-h_{\omega_m^*}[\omega_m^*,\ \psi(\omega_m^*)]\}}=\frac{v_2'[c_f^2(\omega_f^*)]\{c_f^2(\omega_f^*)'-h_{\omega_f^*}[\phi(\omega_f^*),\ \omega_f^*]\}}{v_1'(y_f-\omega_f^*)-v_2'[c_f^2(\omega_f^*)]h_{\omega_f^*}[\phi(\omega_f^*),\ \omega_f^*]}. \tag{24}$$

Along the Pareto-efficient frontier, equation (24) equates the relative marginal utility of premarital investments to its disutility. When combined with the endowment constraint, equation (20), the first-order conditions of equations (22) and (23) determine the Pareto-efficient frontier. Along this frontier, the wife's utility constraint, equation (19), ties down the allocation associated with the wife attaining utility equal to $\bar{U}_f$.

If couples make their premarital investments efficiently (and given the spousal matching process described in Section 4), then we shall be able to link how intra-household allocations are influenced by spousal endowments. To demonstrate the efficiency of the marriage market outcomes requires showing that these outcomes satisfy equations (20)–(23) and thus lie along the Pareto-efficient frontier. As we prove in our next and final proposition, this is easily done:

Proposition 3. *For all couples* (y_m, y_f), *both the level of premarital investments,* ω_m^* *and* ω_f^*, *and intra-marital spousal allocations,* $c_m^2(\omega_m^*)$ *and* $c_f^2(\omega_f^*)$, *are Pareto efficient.*

Proof. First, note that the resource constraint of equation (20) is implicit in the construction of the marriage market outcomes. Second, the first-order conditions for optimal investment in the marriage market model (equations (15) and (16)) are equivalent to equations (22) and (23). Therefore, the marriage market outcomes are unconditionally efficient.[21] ‖

In Figure 3, we illustrate the equilibrium in terms of premarital investments (*i.e.* as in the top panel of Figure 2, suppressing and treating as implicit the accompanying changes in intra-marital

21. Alternatively, note that equation (24) satisfies all the four equations for unconditional efficiency: (9), (10), (15), and (16).

spousal allocations upon marriage). We superimpose the loci of the Pareto-efficient frontier and the reservation utilities on the curve that shows the equilibrium combinations of premarital investments, the latter which was originally depicted in the top panel of Figure 2.

7. SPOUSAL INPUTS AS PERFECT SUBSTITUTES

So far, we have examined the case in which spousal inputs are complements in marital production and the production of marital goods is characterized by a supermodular production function that satisfies Definition 1. In this section, we present the case in which spousal inputs are substitutes. Doing so demonstrates that the main results we derived above apply more generally to cases in which couples specialize by production activity. It also shows that, in such cases, singles on the long side of the market can attain higher education levels compared with their married counterparts and even some members of the opposite sex with similar endowments.

Definition 4. For a couple with the premarital investment levels of ω_m^* and ω_f^*, $h = h(\omega_m^*, \omega_f^*)$ defines the marital production function where $h = h(\omega_m^*, \omega_f^*)$ is twice differentiable and submodular. Hence, $\forall \omega_m^*,\ \omega_f^* > 0,\ h_{\omega_m^*}(\omega_m^*, \omega_f^*),\ h_{\omega_f^*}(\omega_m^*, \omega_f^*) > 0,\ h_{\omega_m^* \omega_m^*}(\omega_m^*, \omega_f^*)$, $h_{\omega_f^* \omega_f^*}(\omega_m^*, \omega_f^*) < 0$, and $h_{\omega_m^* \omega_f^*}(\omega_m^*, \omega_f^*) < 0$.

Lemma 4. *Given Definition* 4, *negative assortative matching by spousal investment levels* ω_m^* *and* ω_f^* *is optimal in the marriage market.*

Proof. See Appendix. ||

We begin by noting that Lemma 2 still holds and the behaviour of single men and women remains unaltered. Furthermore, the condition for stability in marital assignments given by equation (6) continues to apply. Due to negative sorting in the marriage market, however, the main difference now is that the highest endowment individuals on the long side of the market will remain single. Hence, while Definition 2 still characterizes the marriage market equilibrium, we need to modify Part 1 of the definition as follows:

1′(a). If $r = 1$, then $1 - \hat{G}(\omega_m^*) = \hat{H}(\omega_f^*)$ or, equivalently, $\hat{G}(\omega_m^*) = 1 - \hat{H}(\omega_f^*)$;
1′(b). If $r > 1$, then $r\hat{G}(\omega_m^*) = 1 - \hat{H}(\omega_f^*)$;
1′(c). If $r < 1$, then $r[1 - \hat{G}(\omega_m^*)] = \hat{H}(\omega_f^*)$.

These new conditions state that men (women) with the highest premarital investment levels will remain single when $r > 1$ ($r < 1$).

Given these restrictions, it is straightforward to derive the analogues of equations (7) and (8), which specify the marital matching functions $\omega_m^* = \phi(\omega_f^*)$ and $\omega_f^* = \psi(\omega_m^*)$, although we skip this step in the interest of brevity. Since Parts 2 and 3 of the marriage market equilibrium definition still apply, the matching functions derived from Parts 1′(a)–(c) yield the first-order conditions in (9) and (10).

These first-order conditions again lead to equations (12) and (13) with the exception stemming from the way the terms k and k' are defined: if $r > 1$ so that men are on the long side of the marriage market, then $k = h(\omega_m^s, 0) - \int_{\Omega_m^{\min}}^{\Omega_m^{\max}} h_{\omega_m^*}[s, \psi(s)]ds$ and $k' = h(\Omega_m^{\max}, \Omega_f^{\min}) + g - k$. If $r < 1$ so that women are on the long side of the marriage market, then $k' = h(0, \omega_f^s) - \int_{\Omega_f^{\min}}^{\Omega_f^{\max}} h_{\omega_f^*}[\phi(t), t]dt$ and $k = h(\Omega_m^{\min}, \Omega_f^{\max}) + g - k'$. But if $r = 1$ so that all individuals marry, then all we can infer is that $k + k' = g + h(\Omega_m^{\max}, \Omega_f^{\min}) - \int_{\Omega_m^{\min}}^{\Omega_m^{\max}} h_{\omega_m^*}[s, \psi(s)]ds = g + h(\Omega_m^{\min}, \Omega_f^{\max}) - \int_{\Omega_f^{\min}}^{\Omega_f^{\max}} h_{\omega_f^*}[\phi(t), t]dt$.

Note that the constants k and k' reveal some interesting facts regarding how the household allocations are made between the spouses in general and those in marriages on the boundary of singles in particular. Take the case in which there are excess women, for example, $r < 1$. In this scenario all men are married, and the last married woman, who has the highest endowment among married women, gets $k' + \int_{\Omega_f^{\min}}^{\Omega_f^{\max}} h_{\omega_f^*}[\phi(t), t]dt$. Due to marriage market competition with single women, such a wife is indifferent between staying single and getting married. That is, $h(0, \omega_f^s) = k' + \int_{\Omega_f^{\min}}^{\Omega_f^{\max}} h_{\omega_f^*}[\phi(t), t]dt$. This implies that, while such a wife surrenders all the gains from her marriage to her husband, her utility still exceeds those of women with investment levels below her because of her higher level of premarital investment. Now consider the case in which the sex ratio is equal to unity, and all men and women marry ($r = 1$). While we cannot determine the levels of k and k' separately, we know that their sum must equal the surplus of the marriage net of the consumption of the spouse with the higher endowment according to his or her marginal product. When the husband has the highest premarital investment and the wife the lowest, this comes out to $k + k' = g + h(\Omega_m^{\max}, \Omega_f^{\min}) - \int_{\Omega_m^{\min}}^{\Omega_m^{\max}} h_{\omega_m^*}[s, \psi(s)]ds$. When the wife has the highest premarital investment and the husband the lowest, it equals $k + k' = g + h(\Omega_m^{\min}, \Omega_f^{\max}) - \int_{\Omega_f^{\min}}^{\Omega_f^{\max}} h_{\omega_f^*}[\phi(t), t]dt$. Implicit in these derivations is the fact that if $h(\Omega_m^{\max}, \Omega_f^{\min}) > h(\Omega_m^{\min}, \Omega_f^{\max})$, then the husband with the premarital investment level $\Omega_m^{\max}$ gets a higher amount of the marital output $h(\Omega_m^{\max}, \Omega_f^{\min})$ than the amount the wife with the premarital investment level $\Omega_f^{\max}$ gets from the output of her marriage $h(\Omega_m^{\min}, \Omega_f^{\max})$. In particular, since $k + k'$ is constant, $\int_{\Omega_m^{\min}}^{\Omega_m^{\max}} h_{\omega_m^*}[s, \psi(s)]ds > \int_{\Omega_f^{\min}}^{\Omega_f^{\max}} h_{\omega_f^*}[\phi(t), t]dt$ if $h(\Omega_m^{\max}, \Omega_f^{\min}) > h(\Omega_m^{\min}, \Omega_f^{\max})$. Of course, a similar logic applies when $h(\Omega_m^{\max}, \Omega_f^{\min}) < h(\Omega_m^{\min}, \Omega_f^{\max})$.

Next, it is straightforward to verify that equations (13)–(16) continue to hold. Consequently, Definition 3 holds after making the following modification:

Definition 3′. The investments, $w_m^*(y_m)$ and $w_f^*(y_f)$, and allocations in marriage, $c_m^2[\omega_m^*(y_m)]$ and $c_f^2[\omega_f^*(y_f)]$, define a marriage market equilibrium if there exist endowments, y_m and y_f, for individuals in F and M, respectively, such that

1′(a). If $r = 1$, then $1 - \hat{G}(\omega_m^*(y_m)) = \hat{H}(\omega_f^*(y_f))$ or equivalently $\hat{G}(\omega_m^*(y_m)) = 1 - \hat{H}(\omega_f^*(y_f))$;
1′(b). If $r > 1$, then $r\hat{G}(\omega_m^*(y_m)) = 1 - \hat{H}(\omega_f^*(y_f))$;
1′(c). If $r < 1$, then $r[1 - \hat{G}(\omega_m^*(y_m))] = \hat{H}(\omega_f^*(y_f))$;
2. $\forall \omega_m^*(y_m) \in [\Omega_m^{\min}, \Omega_m^{\max}], c_m^2(\omega_m^*(y_m)) = g + \arg\max_{\omega_f^*}[h(\omega_m^*(y_m), \omega_f^*(y_f)) - c_f^2(\omega_f^*(y_f))]$;
3. $\forall \omega_f^*(y_f) \in [\Omega_f^{\min}, \Omega_f^{\max}], c_f^2(\omega_f^*(y_f)) = g + \arg\max_{\omega_m^*}[h(\omega_m^*(y_m), \omega_f^*(y_f)) - c_m^2(\omega_m^*(y_m))]$;
4. Investments and consumption decisions satisfy the first-order conditions of equations (15) and (16).

We conclude by making two observations: first, the Pareto-efficient levels of premarital investments and intra-household allocations are still characterized by the first-order conditions in (22) and (23). Taken together with the fact that (15) and (16) apply here as well, Proposition 3 still holds so that all choices are Pareto efficient. Second, it is straightforward to verify that single individuals on the margin invest more than their married counterparts when spousal inputs are substitutes. This stands in contrast to the case in which inputs are complements and the pre-marital investment levels of the last couple in the positive assortative rank exceed those who remain single.

8. EXAMPLES

To illustrate our main findings, we now present some examples. In the first one, we provide an analytical solution to a simple specification that enables us to derive closed-form solutions. We use this example to verify the main findings we presented in Sections 3–6. In our second example, we switch to a more general specification that can only be solved numerically. We use this example to carry out comparative static analyses. In our third and final example, we present a version of our model where spousal inputs are substitutes as a result of which negative assortative matching holds in equilibrium as described in Section 7.

(I) Suppose that the distribution of endowments, $y_i, i = f, m$, is uniform on $[y^{\min}, y^{\max}]$ with $y^{\min} > 1$. For simplicity, let the marital gain, g, equal 0 so that $k = k' = 0$, and let the marital production function be given by

$$h(\omega_m, \omega_f) = (\omega_m \omega_f)^{1/2}. \tag{25}$$

Suppose that the preferences of males and females are represented by the following inter-temporal utility function, respectively:

$$U = u_1(c_m^1) + u_2(c_m^2) = (c_m^1)^{1/2} + c_m^2 \tag{26}$$

and

$$V = \upsilon_1(c_f^1) + \upsilon_2(c_f^2) = (c_f^1)^{1/2} + c_f^2. \tag{27}$$

The first term in each equation, respectively, corresponds to $u_1(y_m - \omega_m)$ and $\upsilon_1(y_f - \omega_f)$, and the second terms together correspond to equations (11) and (12).

We can now express the consumption shares of husbands and wives during marriage, $c_m^2(\omega_m^*)$ and $c_f^2(\omega_f^*)$, as follows:

$$c_m^2 = \frac{1}{2} \int_{\omega_m^0}^{\omega_m^*} [\psi(s)/s]^{1/2} ds \tag{28}$$

and

$$c_f^2 = \frac{1}{2} \int_{\omega_f^0}^{\omega_f^*} [\phi(t)/t]^{1/2} dt\ , \tag{29}$$

where

$$\omega_m^0 \begin{cases} = \Omega_m^{\min} & \text{if} \quad r \leq 1 \\ > \Omega_m^{\min} & \text{if} \quad r > 1 \end{cases} \tag{30}$$

and

$$\omega_f^0 \begin{cases} = \Omega_f^{\min} & \text{if} \quad r \geqslant 1 \\ > \Omega_f^{\min} & \text{if} \quad r < 1. \end{cases} \tag{31}$$

Note that the marriage market matching conditions given by equations (11) and (12) are already embedded in the constructs of (28) and (29). Thus, in order to determine the optimal and efficient levels of premarital investments that also yield unconditionally efficient intra-household allocations, we can maximize (26) and (27) with respect to ω_m and ω_f, respectively, and subject to equations (28) through (31). The first-order conditions associated with this problem are given by the following:

$$\frac{1}{(y_m-\omega_m^*)^{1/2}}=\left[\frac{\psi(\omega_m^*)}{\omega_m^*}\right]^{1/2}=\left(\frac{\omega_f^*}{\omega_m^*}\right)^{1/2} \tag{32}$$

and

$$\frac{1}{(y_f-\omega_f^*)^{1/2}}=\left[\frac{\phi(\omega_f^*)}{\omega_f^*}\right]^{1/2}=\left(\frac{\omega_m^*}{\omega_f^*}\right)^{1/2}. \tag{33}$$

Using (32) and (33), we can generate two response functions $\omega_m = f(\omega_f; y_m)$ and $\omega_f = g(\omega_m; y_f)$ as we defined above. These functions are such that, $\forall(\omega_m^*, \omega_f^*)$, they yield the following:

$$\omega_m^* = \frac{y_m\psi(\omega_m^*)}{1+\psi(\omega_m^*)}=\frac{y_m\omega_f^*}{1+\omega_f^*} \quad \text{and} \quad \omega_f = \frac{y_f\phi(\omega_f^*)}{1+\phi(\omega_f^*)}=\frac{y_f\omega_m^*}{1+\omega_m^*}. \tag{34}$$

Using (34), we can solve for the spousal levels of premarital investments for each pair in equilibrium, (ω_m^*, ω_f^*):

$$\omega_m^* = \frac{y_m y_f - 1}{1+y_f} \quad \text{and} \quad \omega_f^* = \frac{y_m y_f - 1}{1+y_m}. \tag{35}$$

Therefore, the uniformity of the endowment distributions over $[y^{\min}, y^{\max}]$ together with (34) and the marriage market clearing condition, $r[1-G(y_m)] = 1-H(y_f)$, implies the following:

$$r\left(1-\frac{y_m - y^{\min}}{y^{\max}-y^{\min}}\right)=1-\frac{y_f - y^{\min}}{y^{\max}-y^{\min}}. \tag{36}$$

Establishing the properties of the equilibrium distributions of premarital investments requires specifying the sex ratio r:

1. If $r = 1$ so that the measures of men and women in the marriage market are identical, all individuals marry. Given equation (35), we can establish that, $\forall(\omega_m^*, \omega_f^*), y_m = y_f$. As a result, equation (34) implies that

$$\omega_m^* = \omega_f^* = y_m - 1 = y_f - 1. \tag{37}$$

   Hence, we establish that when $r = 1$ so that all individuals marry, both spouses get equal shares of the marital output. That is, $\forall(\omega_m^*, \omega_f^*), y_m = y_f \Rightarrow \omega_m^* = \omega_f^* \Rightarrow u_1 = v_1$.
2. If $r > 1$ so that there are fewer women than men in the marriage market, there will be some unmarried men in equilibrium. Using equation (36), we can establish that, $\forall(\omega_m^*, \omega_f^*), y_m = [(r-1)y^{\max} + y_f]/r$. Accordingly, the endowment of the wife in the lowest assortative rank is equal to $y_{\min}$, but that of her husband equals $[(r-1)y^{\max} + y^{\min}]/r > y^{\min}$.
   We can rewrite (35) to determine the endowment matches that emerge in equilibrium when $r > 1$. That is, $\forall(\omega_m^*, \omega_f^*)$,

$$\omega_m^* = \frac{\left[\frac{(r-1)y^{\max}+y_f}{r}\right]y_f - 1}{1+y_f} \quad \text{and} \quad \omega_f^* = \frac{\left[\frac{(r-1)y^{\max}+y_f}{r}\right]y_f - 1}{1+\frac{(r-1)y^{\max}+y_f}{r}}, \tag{38}$$

   where, by construction, $\forall(\omega_m^*, \omega_f^*), \omega_m^* > \omega_f^*$. Hence, we conclude, $\forall(\omega_m^*, \omega_f^*), y_m > y_f \Rightarrow \omega_m^* > \omega_f^* \Rightarrow u_1 > v_1$.

3. If $r < 1$ so that there are more women than men in the marriage market, there will be some unmarried women in equilibrium. Again relying on (36) and the uniformity of the endowment distributions over $[y^{\min}, y^{\max}]$, we can establish that, $\forall(\omega_m^*, \omega_f^*)$, $y_f = [(1-r)y^{\max} + ry_m]$. And similar to the case above, the endowment of the husband in the lowest assortative rank is equal to $y_{\min}$, but that of his wife equals $[(1-r)y^{\max} + ry^{\min}] > y^{\min}$. Following the steps above, we can determine the endowment matches that emerge in equilibrium when $r > 1 : \forall(\omega_m^*, \omega_f^*)$:

$$\omega_m^* = \frac{[ry^{\max} + (1-r)y_m]y_m - 1}{1 + ry^{\max} + (1-r)y_m} \quad \text{and} \quad \omega_f^* = \frac{[ry^{\max} + (1-r)y_m]y_m - 1}{1 + y_m}, \tag{39}$$

where, by construction, $\forall(\omega_m^*, \omega_f^*)$, $\omega_m^* < \omega_f^*$. Hence, we conclude, $\forall(\omega_m^*, \omega_f^*)$, $y_m < y_f \Rightarrow \omega_m^* < \omega_f^* \Rightarrow u_1 < \upsilon_1$.

(II) Next, suppose that the distribution of endowments, $y_i, i = f, m$, is uniform on $[0, 1]$. Let the marital production function be given by

$$h(\omega_m, \omega_f) = \frac{(\omega_m + \omega_f)^2}{2}. \tag{40}$$

Also suppose that the preferences of males and females are represented by the following inter-temporal utility function, respectively:

$$U = u_1(c_m^1) + u_2(c_m^2) = \ln c_m^1 + \ln c_m^2 \tag{41}$$

and

$$V = \upsilon_1(c_f^1) + \upsilon_2(c_f^2) = \ln c_f^1 + \ln c_f^2. \tag{42}$$

(Note that the existence of a stable spousal matching equilibrium for this utility specification is given by Lemma 3.) The first term in each equation, respectively, corresponds to $u_1(y_m - \omega_m)$ and $\upsilon_1(y_f - \alpha\omega_f)$, where $\alpha, 1 \le \alpha$, represents the gender bias in the cost of education. And the second-period consumption levels of husbands and wives are, respectively, given by plugging $h_{\omega_m^*}(\cdot)$ and $h_{\omega_f^*}(\cdot)$ from the assumed form of $h(\cdot)$ given by (40) into equations (11) and (12):

$$c_m^2 = k + \int_{\omega_m^0}^{\omega_m^*} [s + \psi(s)]ds \tag{43}$$

and

$$c_f^2 = k' + \int_{\omega_f^0}^{\omega_f^*} [\phi(t) + t]dt, \tag{44}$$

where

$$\omega_m^0 \begin{cases} = 0 & \text{if} \quad r \le 1 \\ > 0 & \text{if} \quad r > 1 \end{cases} \tag{45}$$

and

$$\omega_f^0 \begin{cases} = 0 & \text{if} \quad r \geqslant 1 \\ > 0 & \text{if} \quad r < 1. \end{cases} \tag{46}$$

Using this example, we explore how changes in the sex ratios in the marriage markets and the gender bias in the cost of education influence this equilibrium.[22]

22. The Matlab code used in generating the results that follow is available from the authors upon request.

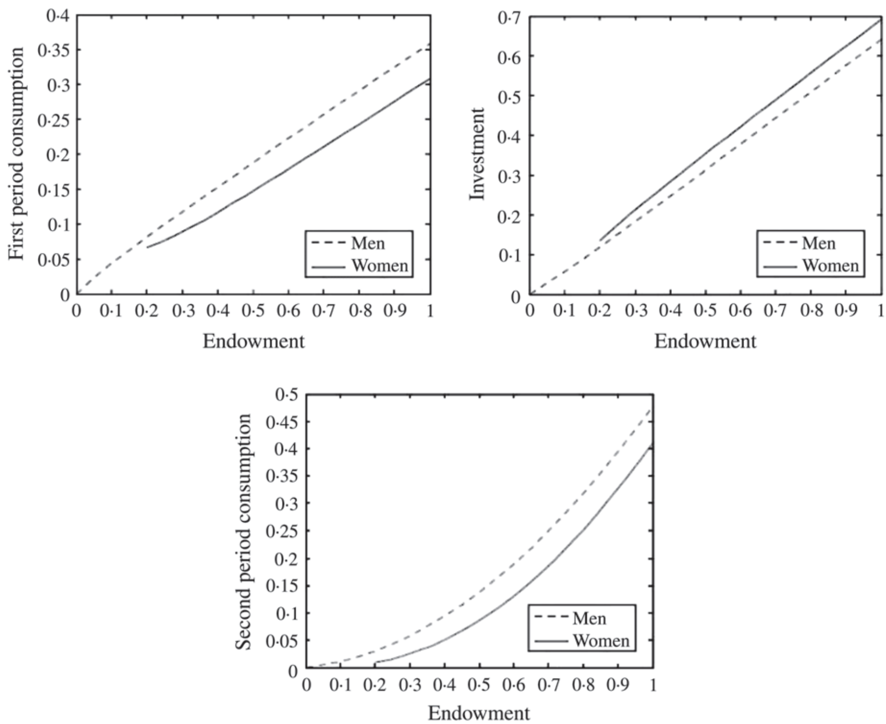

FIGURE 4

Consumption and investment levels, $r = 0{\cdot}80$

Consistent with Lemma 3, we find in all exercises we carry out that the marital matching functions $\phi(\omega_f)$ and $\psi(\omega_m)$ are such that, $\forall(\omega_m^*, \omega_f^*)$, $\omega_m^* = \phi(\omega_f^*)$ and $\omega_f^* = \psi(\omega_m^*)$. That is, in all specifications, our numerical exercise generates marital matching functions that are consistent with a unique equilibrium in the marriage markets.

Figures 4, 5, and Table 1 summarize the results of the numerical exercises with no gender difference in the cost of education (*i.e.* $\alpha = 1$) and when $r = 0{\cdot}80$. Figure 4 compares consumption and investment behaviours, while the four panels of Figure 5 evaluate the effect of marriage on individual utility. The marital surplus measures presented in panel (c) of Figure 5 measure the percentage of the marital surplus consumed by the wife, where we define marital surplus as the aggregate consumption available to the couple in marriage less the sum of singles consumption (holding investment levels constant at their equilibrium levels). In addition to the more interesting asymmetric case, Table 1 presents results for the symmetric case where $r = 1$. In section (a), we present the results for $r = 1$, in section (b), we tabulate those for $r = 0{\cdot}80$. The table is organized based on the endowment of the husband. For each of the two panels, spousal consumption and investment levels, their second-period consumption levels as singles, $(c_i^2)^s, i = f, m$, and the wife's percentage of total surplus (as defined above) are reported.

In the symmetric case in which $r = 1$ and all individuals get married, each couple along the assortative order has spouses with identical endowments, premarital investments, and intra-household allocation shares. As is shown in section (b) of Table 1, a higher ratio of women to men in the marriage markets transfers marital gains from women to men—with the effect being most pronounced among the lower assortative ranks. This transfer arises as women compete away their surplus in response to their increased abundance. As shown in the four panels of Figure 5,

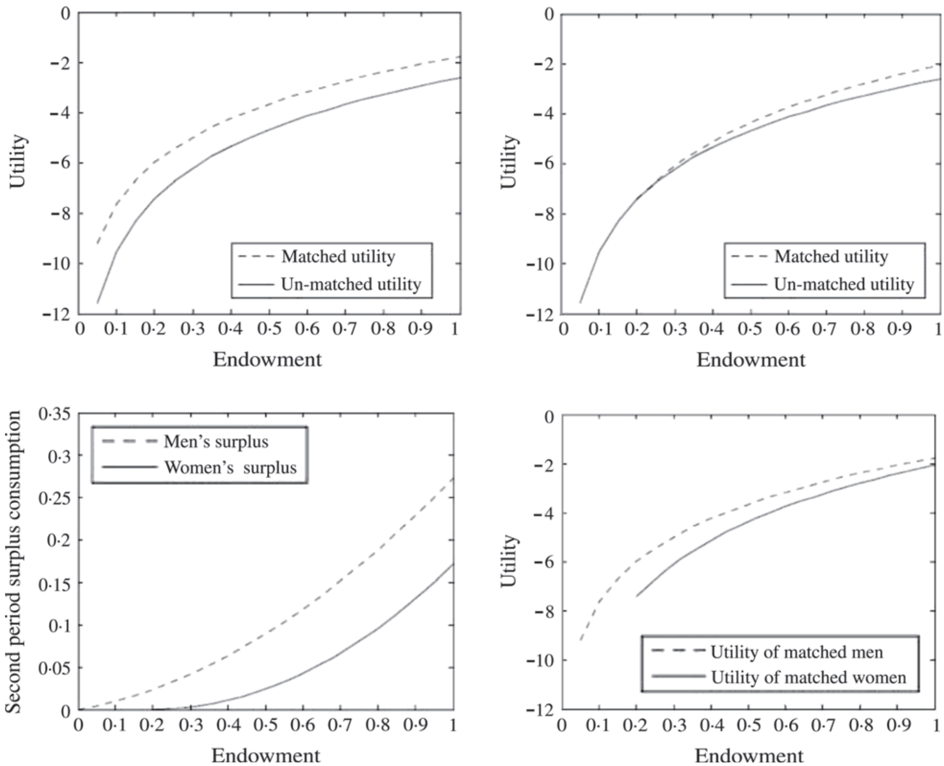

FIGURE 5

Utility and surplus, $r = 0{\cdot}80$

they partially mitigate this new disadvantage through increased investments but still consume less than men of identical endowment levels in both periods. Moreover, as depicted in the top two panels of Figure 5, all married individuals' utility levels exceed those of singles with identical endowment levels. The exceptions to this observation are the married women in the lowest rank who receive their singles (reservation) utility. Since women in the lowest assortative rank marry men whose endowments are equal to 0, and the equilibrium in the marriage markets ensures that all spouses receive the marginal return to their premarital investments, the premarital investments of married, lowest rank women equal those of single women with identical endowment levels. But because women in higher ranks marry men whose endowment levels are more similar to theirs (which raises the wives' marginal return to premarital investments), such women invest a relatively higher share of their endowments compared with women in lower ranks. As shown in the last panel of Figure 5, this implies that women capture an increasing share of the marital gains as their ranks in the assortative order rises. Finally, compared with the case in which the sex ratio is fully balanced ($r = 1$), the optimal premarital investment levels of the more (less) abundant sex increase (decrease) when the sex ratio is not in balance ($r \neq 1$). In particular, it can be verified that women, who are on the long side of the market, invest more than men with identical endowments. In contrast, the optimal second-period consumption levels of the more (less) abundant gender fall (rise) when the sex ratio is not in balance. Both of these observations are due to the fact that the higher the share of the marital gains transferred to the less abundant gender, the more unbalanced is the sex ratio. Nonetheless, the impact of an unbalanced sex ratio is stronger in the lower assortative ranks, and it dissipates as the rank of a couple rises. In fact, as seen in the last

TABLE 1

Changes in the sex ratio, r, and equilibrium investments and allocations

(a) $r = 1$									(b) $r = 0{\cdot}80$							
y_m	y_f	ω_m	ω_f	c_m^2	c_f^2	$(c_m^2)^s$	$(c_f^2)^s$	$\frac{\text{sur } f}{\text{t sur}}$	y_f	ω_m	ω_f	c_m^2	c_f^2	$(c_m^2)^s$	$(c_f^2)^s$	$\frac{\text{sur } f}{\text{t sur}}$
0·00	0·00	0·000	0·000	0·000	0·000	0·000	0·000	—	0·20	0·000	0·133	0·000	0·009	0·000	0·009	—
0·05	0·05	0·033	0·033	0·001	0·001	0·001	0·001	0·500	0·24	0·027	0·166	0·004	0·014	0·000	0·014	0·108
0·10	0·10	0·067	0·067	0·004	0·004	0·002	0·002	0·500	0·28	0·057	0·197	0·011	0·021	0·002	0·019	0·156
0·15	0·15	0·100	0·100	0·010	0·010	0·005	0·005	0·500	0·32	0·087	0·226	0·020	0·029	0·004	0·026	0·195
0·20	0·20	0·133	0·133	0·018	0·018	0·009	0·009	0·500	0·36	0·119	0·255	0·030	0·039	0·007	0·032	0·226
0·25	0·25	0·167	0·167	0·028	0·028	0·014	0·014	0·500	0·40	0·150	0·283	0·043	0·051	0·011	0·040	0·250
0·30	0·30	0·200	0·200	0·040	0·040	0·020	0·020	0·500	0·44	0·182	0·311	0·058	0·064	0·017	0·048	0·268
0·35	0·35	0·233	0·233	0·054	0·054	0·027	0·027	0·500	0·48	0·215	0·339	0·075	0·078	0·023	0·057	0·285
0·40	0·40	0·267	0·267	0·071	0·071	0·036	0·036	0·500	0·52	0·247	0·366	0·094	0·094	0·030	0·067	0·299
0·45	0·45	0·300	0·300	0·090	0·090	0·045	0·045	0·500	0·56	0·280	0·394	0·115	0·112	0·039	0·078	0·312
0·50	0·50	0·333	0·333	0·111	0·111	0·056	0·056	0·500	0·60	0·312	0·421	0·138	0·131	0·049	0·089	0·322
0·55	0·55	0·367	0·367	0·134	0·134	0·067	0·067	0·500	0·64	0·345	0·449	0·163	0·152	0·059	0·101	0·332
0·60	0·60	0·400	0·400	0·160	0·160	0·080	0·080	0·500	0·68	0·378	0·476	0·190	0·174	0·071	0·113	0·340
0·65	0·65	0·433	0·433	0·188	0·188	0·094	0·094	0·500	0·72	0·411	0·503	0·219	0·198	0·084	0·126	0·348
0·70	0·70	0·467	0·467	0·218	0·218	0·109	0·109	0·500	0·76	0·443	0·530	0·250	0·224	0·098	0·140	0·355
0·75	0·75	0·500	0·500	0·250	0·250	0·125	0·125	0·500	0·80	0·476	0·557	0·283	0·251	0·113	0·155	0·362
0·80	0·80	0·533	0·533	0·284	0·284	0·142	0·142	0·500	0·84	0·509	0·584	0·318	0·280	0·130	0·171	0·367
0·85	0·85	0·567	0·567	0·321	0·321	0·161	0·161	0·500	0·88	0·542	0·611	0·355	0·310	0·147	0·187	0·373
0·90	0·90	0·600	0·600	0·360	0·360	0·180	0·180	0·500	0·92	0·575	0·638	0·394	0·342	0·165	0·204	0·377
0·95	0·95	0·633	0·633	0·401	0·401	0·201	0·201	0·500	0·96	0·608	0·665	0·435	0·376	0·185	0·221	0·382
1·00	1·00	0·667	0·667	0·444	0·444	0·222	0·222	0·500	1·00	0·641	0·692	0·478	0·411	0·206	0·239	0·386

n.m., not married; t sur, total surplus.

TABLE 2

Changes in the gender gap, α, and equilibrium investments and allocations

	(a) $r = 0{\cdot}80, \alpha = 1{\cdot}2$								(b) $r = 0{\cdot}80, \alpha = 1{\cdot}1$							
y_m	y_f	ω_m	ω_f	c_m^2	c_f^2	$(c_m^2)^s$	$(c_f^2)^s$	$\frac{\text{sur } f}{\text{t sur}}$	y_f	ω_m	ω_f	c_m^2	c_f^2	$(c_m^2)^s$	$(c_f^2)^s$	$\frac{\text{sur } f}{\text{t sur}}$
0·000	0·20	0·000	0·111	0·000	0·006	0·000	0·006	—	0·20	0·000	0·121	0·000	0·007	0·000	0·007	—
0·050	0·24	0·027	0·139	0·004	0·010	0·000	0·010	0·101	0·24	0·027	0·152	0·004	0·012	0·000	0·011	0·099
0·100	0·28	0·057	0·165	0·010	0·015	0·002	0·014	0·153	0·28	0·057	0·180	0·010	0·018	0·002	0·016	0·155
0·150	0·32	0·088	0·190	0·017	0·021	0·004	0·018	0·195	0·32	0·088	0·206	0·018	0·025	0·004	0·021	0·194
0·200	0·36	0·119	0·214	0·027	0·029	0·007	0·023	0·227	0·36	0·119	0·233	0·029	0·033	0·007	0·027	0·226
0·250	0·40	0·151	0·238	0·038	0·037	0·011	0·028	0·249	0·40	0·151	0·258	0·041	0·043	0·011	0·033	0·250
0·300	0·44	0·183	0·261	0·052	0·047	0·017	0·034	0·268	0·44	0·183	0·284	0·055	0·054	0·017	0·040	0·268
0·350	0·48	0·216	0·284	0·067	0·058	0·023	0·040	0·286	0·48	0·215	0·309	0·071	0·067	0·023	0·048	0·285
0·400	0·52	0·248	0·307	0·084	0·070	0·031	0·047	0·300	0·52	0·248	0·334	0·089	0·081	0·031	0·056	0·299
0·450	0·56	0·281	0·330	0·103	0·083	0·039	0·055	0·312	0·56	0·280	0·359	0·109	0·096	0·039	0·064	0·312
0·500	0·60	0·314	0·353	0·124	0·098	0·049	0·062	0·322	0·60	0·313	0·384	0·130	0·113	0·049	0·074	0·323
0·550	0·64	0·347	0·376	0·147	0·114	0·060	0·071	0·332	0·64	0·346	0·409	0·154	0·131	0·060	0·084	0·332
0·600	0·68	0·379	0·399	0·172	0·131	0·072	0·079	0·340	0·68	0·379	0·434	0·180	0·150	0·072	0·094	0·340
0·650	0·72	0·412	0·421	0·198	0·149	0·085	0·089	0·348	0·72	0·411	0·458	0·208	0·171	0·085	0·105	0·348
0·700	0·76	0·445	0·444	0·227	0·169	0·099	0·098	0·355	0·76	0·444	0·483	0·237	0·193	0·099	0·117	0·355
0·750	0·80	0·478	0·466	0·257	0·189	0·114	0·109	0·361	0·80	0·477	0·508	0·269	0·216	0·114	0·129	0·361
0·800	0·84	0·511	0·489	0·289	0·211	0·131	0·120	0·367	0·84	0·510	0·532	0·302	0·241	0·130	0·142	0·367
0·850	0·88	0·544	0·511	0·323	0·234	0·148	0·131	0·372	0·88	0·543	0·557	0·337	0·268	0·148	0·155	0·372
0·900	0·92	0·577	0·534	0·359	0·259	0·167	0·143	0·377	0·92	0·576	0·581	0·375	0·295	0·166	0·169	0·377
0·950	0·96	0·610	0·556	0·396	0·284	0·186	0·155	0·381	0·96	0·609	0·606	0·414	0·324	0·186	0·183	0·382
1·000	1·00	0·643	0·579	0·436	0·311	0·207	0·167	0·386	1·00	0·642	0·630	0·455	0·355	0·206	0·199	0·386

n.m., not married; t sur, total surplus.

columns of section (b), wives' share of the marital surplus monotonically increases as a couple's marital rank rises.

This last observation is quite important with regard to the arguments of intra-household sharing rules: while both the marriage market sex ratio and relative spousal endowments influence intra-household allocations and premarital investments, the relative importance of the sex ratio increases and that of relative spousal endowments decreases as the rank of a couple in the assortative order increases.

Next, we investigate how the gender gap in the cost of education would affect equilibrium investments and spousal allocations. Section (a) of Table 2 presents the equilibrium outcomes when there are more women than men in the marriage market with $r = 0{\cdot}80$ and the unit cost of acquiring education is 20% higher for women so that $\alpha = 1{\cdot}2$. Thus, compared with section (a) of Table 1, the fact that the cost of education is higher for women lowers their incentives to invest in education, but the higher abundance of women in the marriage markets raises them. In this case, the cost gap is still large enough that women along the assortative order invest less than they do in Table 1, section (a). This verifies that the cost-gap effect dominates the sex-ratio effect. The higher cost of education for women is also detrimental for men's investment levels. In particular, when $r < 1$, the sex ratio is skewed in men's favour as a result of which they invest less and consume more (as shown in section (b) of Table 1). Nonetheless, the higher cost of education for women leaves them worse off than men to whom they are married because their intra-household allocations have declined and their premarital investments have risen. As can be verified by the comparisons of the surplus women get in their marriages in section (b) of Table 1 and section (a) of Table 2, the detrimental impact of the higher cost of education on women's intra-marital welfare is higher at lower ranks than it is at higher assortative[1 levels.]

In section (b) of Table 2, we show the outcomes when the sex ratio r is still 0·80, but the cost of education for women is only 10% higher than it is for men so that α equals 0·10. When women's cost of education is still high relative to men, as it is in this section, men invest even less than they do in section (b) of Table 1 because their marginal contribution to marital output declines with decreases in women's education. Note that, under this scenario, women all along the assortative matching order invest more than men with identical endowment levels despite the fact that education is more costly for women. This highlights the impact of the sex ratio on equilibrium investment levels. In addition, all (married) men undertake more premarital investments than they do when women's education cost is higher (as it is in section (a) of Table 2), reflecting the higher marginal contribution of men to marital output due to the increased investment levels of their wives.

(III) Finally, we consider an example in which spousal attributes are substitutes in marital production instead of complements. As before, we assume that the distribution of endowments, $y_i, i = f, m,$ is uniform on [0, 1], but we let the marital production function be given by

$$h(\omega_m, \omega_f) = 2(\omega_m + \omega_f)^{1/2} + g; \quad g = 0{\cdot}50. \tag{47}$$

Note that the substitution between the spousal inputs requires us to specify a strictly positive marital gain, g, in this case. Given the support of the endowment levels for men and women, $g = 0{\cdot}50$ ensures that the participation constraints for all married individuals are satisfied (*i.e.* that all married individuals are at least as well off in marriage as they are as singles).

The preferences of men and women are represented by equations (41) and (42). Recall that the first term in each of those equations, respectively, corresponds to $u_1(y_m - \omega_m)$ and $\upsilon_1(y_f - \alpha\omega_f)$ and the second-period consumption levels of husbands and wives are, respectively, given by plugging $h_{\omega_m^*}(\cdot)$ and $h_{\omega_f^*}(\cdot)$ from the assumed form of $h(\cdot)$, now given by (47)

TABLE 3

Spousal inputs as substitutes

	(a) $r = 1$								(b) $r = 0{\cdot}80$							
y_m	y_f	ω_m	ω_f	c_m^2	c_f^2	$(c_m^2)^s$	$(c_f^2)^s$	$\frac{\text{sur}\,f}{\text{t sur}}$	y_f	ω_m	ω_f	c_m^2	c_f^2	$(c_m^2)^s$	$(c_f^2)^s$	$\frac{\text{sur}\,f}{\text{t sur}}$
0·00	1·00	0·000	0·333	0·500	1·154	0·000	1·154	0·000	0·80	0·000	0·267	0·500	1·033	0·000	1·033	0·000
0·05	0·95	0·000	0·316	0·500	1·125	0·000	1·125	0·000	0·76	0·000	0·253	0·500	1·006	0·000	1·006	0·000
0·10	0·90	0·000	0·300	0·500	1·095	0·000	1·095	0·000	0·72	0·000	0·240	0·500	0·980	0·000	0·980	0·000
0·15	0·85	0·000	0·283	0·500	1·064	0·000	1·064	0·000	0·68	0·000	0·227	0·500	0·952	0·000	0·952	0·000
0·20	0·80	0·000	0·267	0·500	1·033	0·000	1·033	0·000	0·64	0·000	0·213	0·500	0·924	0·000	0·924	0·000
0·25	0·75	0·000	0·250	0·500	1·000	0·000	1·000	0·000	0·60	0·011	0·196	0·524	0·886	0·212	0·886	0·002
0·30	0·70	0·025	0·225	0·550	0·950	0·316	0·949	0·006	0·56	0·035	0·175	0·577	0·840	0·376	0·836	0·016
0·35	0·65	0·050	0·200	0·600	0·900	0·447	0·894	0·035	0·52	0·059	0·154	0·630	0·794	0·487	0·784	0·063
0·40	0·60	0·075	0·175	0·650	0·850	0·548	0·837	0·115	0·48	0·083	0·133	0·681	0·748	0·578	0·728	0·161
0·45	0·55	0·100	0·150	0·700	0·800	0·632	0·775	0·273	0·44	0·107	0·111	0·733	0·703	0·655	0·668	0·311
0·50	0·50	0·125	0·125	0·750	0·750	0·707	0·707	0·501	0·40	0·131	0·090	0·784	0·658	0·724	0·601	0·490
0·55	0·45	0·150	0·100	0·800	0·700	0·775	0·632	0·727	0·36	0·155	0·069	0·834	0·613	0·787	0·527	0·650
0·60	0·40	0·175	0·075	0·850	0·650	0·837	0·548	0·885	0·32	0·179	0·049	0·884	0·570	0·845	0·440	0·771
0·65	0·35	0·200	0·050	0·900	0·600	0·894	0·447	0·965	0·28	0·202	0·028	0·933	0·526	0·900	0·333	0·851
0·70	0·30	0·225	0·025	0·950	0·550	0·949	0·316	0·995	0·24	0·226	0·007	0·982	0·483	0·951	0·166	0·909
0·75	0·25	0·250	0·000	1·000	0·500	1·000	0·000	1·000	0·20	0·245	0·000	1·021	0·469	0·989	0·000	0·937
0·80	0·20	0·267	0·000	1·033	0·500	1·033	0·000	1·000	0·16	0·261	0·000	1·054	0·469	1·022	0·000	0·937
0·85	0·15	0·283	0·000	1·064	0·500	1·064	0·000	1·000	0·12	0·278	0·000	1·085	0·469	1·054	0·000	0·937
0·90	0·10	0·300	0·000	1·095	0·500	1·095	0·000	1·000	0·08	0·294	0·000	1·117	0·469	1·085	0·000	0·937
0·95	0·05	0·316	0·000	1·125	0·500	1·125	0·000	1·000	0·04	0·311	0·000	1·147	0·469	1·115	0·000	0·937
1·00	0·00	0·333	0·000	1·154	0·500	1·154	0·000	1·000	0·00	0·327	0·000	1·176	0·469	1·144	0·000	0·937

n.m., not married; t sur, total surplus.

into equations (11) and (12):

$$c_m^2 = k + \int_{\omega_m^0}^{\omega_m^*} [s + \psi(s)]^{-1/2} ds \tag{48}$$

and

$$c_f^2 = k' + \int_{\omega_f^0}^{\omega_f^*} [\phi(t) + t]^{-1/2} dt, \tag{49}$$

where k, k' as well as the identity of individuals who remain single on the long side of the market, $\Omega_f^{\max}$ and $\Omega_m^{\max}$, are determined as discussed in Section 7.

In Table 3, we do away with differences in the cost of education so that $\alpha = 1$ and present the equilibrium outcomes when $r = 1$ and $r = 0{\cdot}80$. There are a number of salient contrasts with the results we presented above. First, note that all individuals invest less due to the fact that spousal inputs are substitutes. Second, all single individuals invest more than their married counterparts after controlling for their endowments. And third, many married individuals with low endowments choose not to invest at all but, when an interior solution exists, the endowment elasticity of premarital investments is higher than in our previous examples in Table 1. The latter produces more heterogeneity both among all individuals of a given gender and married individuals of a specific gender.

9. CONCLUSION

In recent years, the "collective" model of the household, in which individual members of the family are treated as the core decision-makers and a sharing rule generates efficient intra-household allocations, has emerged as the most promising framework for understanding household behaviour. These models suggest that relative spousal incomes influence household allocations, but they do not account for the fact that the household income can be determined at least in part by decisions individuals make prior to marriage. As we point out above, the lack of an integrated theoretical framework—in which the intra-household sharing rules are derived endogenously—accounts for why empirical studies in this area remain agnostic with respect to the factors that could determine intra-marital allocations.

In this paper, we present the first attempt to extend the collective household model to cover premarital investments and matching in the marriage markets. Our endeavour shows that, for each couple, an endogenously determined sharing rule emerges as the only maritally sustainable outcome. For each couple, the equilibrium sharing rule (i) yields Pareto-efficient investment choices and intra-household spousal allocations, and (ii) is a function of the sex ratio in the marriage market and the (premarital) distribution of spousal endowments.

According to our results, transfers among spouses occur on two margins: premarital investments and intra-marital spousal allocations. Asymmetries in the sex ratios in the marriage markets produce gender differences in premarital investments and consumption that are larger for individuals with small premarital endowments than those with larger endowments. Furthermore, the sex ratio has a larger impact than relative spousal endowments on intra-household allocations higher up in the marital assortative ranks.

Endogenizing the intra-marital sharing rules in conjunction with premarital investments allows us to provide explanations for some empirical regularities. For example, because individuals on the long side of the market and in lower-ranked marriages compete more intensely for their scarce mates (than richer individuals on the same side of the market), they surrender more of their marital surplus to their spouses. An important corollary of this is that, when men are in short supply in the marriage markets, women can invest more than men premaritally even when the returns to their investment are lower or the costs are higher for women.

In developing countries, premarital investments often take the form of dowries and bride prices, and there is some evidence that changes in dowries and bride prices can be attributed to changes in the sex ratios in the marriage markets. Our model not only yields such a link but also shows that increases in dowries also ought to be accompanied by lower intra-marital allocations for wives of lower marriage ranks.

Finally, a narrower gender wage gap or reductions in the cost of education induce women to invest relatively more regardless of the marital production technology. However we find that, if spousal attributes are complements and there are more women than men, women in the lower ranks would invest relatively more than those in higher ranks. Moreover, regardless of their gender, singles would invest less than similarly endowed married individuals. But if spousal attributes are substitutes and there are more women than men in the marriage markets, single women, who would be those at the top of the female endowment distribution, would be more educated than their married counterparts.

APPENDIX

Proof of Lemma 1. According to Definition 1, the marital production technology $h(\omega_m^*, \omega_f^*)$ is supermodular. As a result, if we take two men with the investment levels of ω_m and ω_m' such that $\omega_m' > \omega_m$ and two women with ω_f and ω_f' where $\omega_f' > \omega_f$, then by definition it has to be the case that

$$h(\omega_m', \omega_f') + h(\omega_m, \omega_f) > h(\omega_m', \omega_f) + h(\omega_m, \omega_f'). \tag{A.1}$$

Equation (A.1) implies that

$$h(\omega_m', \omega_f') - h(\omega_m', \omega_f) > h(\omega_m, \omega_f') - h(\omega_m, \omega_f). \tag{A.2}$$

Given (A.2), the man with the investment level ω_m' gains more from marrying the woman with the investment level ω_f' than the man with ω_m. And since utility is fully transferable between spouses, the man with ω_m' can outbid the man with ω_m in the competition to marry the woman with ω_f'. As a result, the pairings (ω_m', ω_f') and (ω_m, ω_f) are maritally sustainable, while the pairings (ω_m', ω_f) and (ω_m, ω_f') are not. ‖

Proof of Proposition 1. Our proof is a direct application of Gretsky *et al.* (1992). In a one-to-one assignment game, they prove that (i) the game has a solution and (ii) it corresponds to the Walrasian equilibrium as long as (a) the population measure on each side of the market is defined over a compact metric space, (b) the valuation function is upper semi-continuous, and (c) transfers are allowed between the two sides of the market.

Since $\forall \omega_m^*, \omega_f^* \in [\Omega_m^{\min}, \Omega_m^{\max}]$, our spousal investment distributions are defined over two compact sets. Given Definition 1, the marital production technology $h(\omega_m, \omega_f)$ satisfies upper semi-continuity. And by construction, we consider a transferable utility model. Thus, conditional on the spousal distribution of premarital investments, there exist marriage market equilibria. Regardless of the value of r, equation (7) or (8) matches each man (or woman) to a unique spouse. Given equations (11) and (12), there can be a continuum of equilibrium intra-household allocations for each couple when $r = 1$ but, when $r \neq 1$, equations (11) and (12) define unique intra-household allocations for all couples. Consequently, the stable marriage market equilibrium is unique when $r \neq 1$. ‖

Proof of Proposition 2. The proof of existence requires us to demonstrate that the rational expectations marital matching function defined in equation (8), $\psi(\omega_m^*)$, produces the husbands' premarital investments ω_m^* (or, alternatively, that the rational expectations marital matching function defined in equation (7), $\phi(\omega_f^*)$, produces the wives' premarital investments ω_f^*).

For each $\hat{y}_m$ in the support of G, define

$$\gamma(y_m) = \{\hat{y}_f \in \text{supp } H : 1 - H(\hat{y}_f) = r[1 - G(\hat{y}_m)]\}. \tag{A.3}$$

Given that $U = u_1(c_m^1) + u_2(c_m^2)$ and $V = \upsilon_1(c_f^1) + \upsilon_2(c_f^2)$, the first-order conditions for premarital investments specified in (15) and (16) and the invertibility of the functions $u_1(\cdot)$ and $\upsilon_1(\cdot)$ imply

$$\hat{y}_m = \omega_m^* + (u_1')^{-1}\{u_2'(c_m^2)h_{\omega_m^*}[\omega_m^*, \psi(\omega_m^*)]\} \tag{A.4}$$

and

$$\hat{y}_f = \omega_f^* + (\upsilon_1')^{-1}\{\upsilon_2'(c_\upsilon^2)h_{\omega_\upsilon^*}[\omega_m^*, \psi(\omega_m^*)]\}. \tag{A.5}$$

Equilibrium exists if $\forall \hat{y}_m, \hat{y}_f \in [y^{\min}, y^{\max}], \exists \psi(\omega_m^*)\ s.t.\ \omega_f^* = \psi(\omega_m^*)$. Hence, for $\omega_f^* = \psi(\omega_m^*)$, equilibrium exists if the following equality is satisfied:

$$\begin{aligned} &\gamma\left\{\omega_m^* + (u_1')^{-1}\left[u_2'\left(k + \int_{\omega_m^0}^{\omega_m^*} h_{\omega_m^*}[s, \psi(s)]ds\right)h_{\omega_m^*}[\omega_m^*, \psi(\omega_m^*)]\right]\right\} \\ &= \psi(\omega_m^*) + (\upsilon_1')^{-1}\left[\upsilon_2'\left(h[\omega_m^*, \psi(\omega_m^*)] - k - \int_{\omega_m^0}^{\omega_m^*} h_{\omega_m^*}[s, \psi(s)]ds\right)h_{\omega_\upsilon^*}[\omega_m^*, \psi(\omega_m^*)]\right]. \end{aligned} \tag{A.6}$$

To complete our proof, we need to establish that $\omega_f^* = \psi(\omega_m^*)$ and that it is strictly monotonically increasing in ω_m^*. While we cannot identify regularity assumptions under which this condition for the existence of an equilibrium will be met universally, it is quite straightforward to demonstrate, as we do in Lemma 3, existence for certain utility specifications.

When the function $\omega_f^* = \psi(\omega_m^*)$ exists, and it is strictly monotonically increasing in ω_m^*, the equilibrium allocations will be unique when $r \neq 1$ because spousal allocations described by (11) and (12), c_m^2 and c_f^2, are unique when $r \neq 1$. ‖

Proof of Lemma 3. When $U = u_1(c_m^1) + u_2(c_m^2) = \ln(c_m^2) + \ln(c_m^2)$ and $V = \upsilon_1(c_f^1) + \upsilon_2(c_f^2) = \ln(c_f^2) + \ln(c_f^2)$, the first-order conditions for premarital investments specified in (15) and (16) imply

$$\frac{1}{\hat{y}_m - \omega_m^*} = \frac{h_{\omega_m^*}[\omega_m^*, \psi(\omega_m^*)]}{c_m^2}, \tag{A.7}$$

and,

$$\frac{1}{\hat{y}_f - \omega_f^*} = \frac{h_{\omega_f^*}[\omega_m^*, \psi(\omega_m^*)]}{c_f^2}. \tag{A.8}$$

By solving (A.7) and (A.8) for $\hat{y}_m$ and $\hat{y}_f$ respectively, we get

$$\hat{y}_m = \frac{c_m^2}{h_{\omega_m^*}[\omega_m^*, \psi(\omega_m^*)]} + \omega_m^* \tag{A.9}$$

and

$$\hat{y}_f = \frac{c_f^2}{h_{\omega_f^*}[\omega_m^*, \psi(\omega_m^*)]} + \psi(\omega_m^*). \tag{A.10}$$

Equilibrium exists if $\forall \hat{y}_m, \hat{y}_f \in [y^{\min}, y^{\max}], \exists \psi(\omega_m^*)\, s.t.\, \omega_f^* = \psi(\omega_m^*)$. Hence, for $\omega_f^* = \psi(\omega_m^*)$, equilibrium exists if the following equality is satisfied:

$$\gamma\left[\frac{k + \int_{\omega_m^0}^{\omega_m^*} h_{\omega_m^*}[s, \psi(s)]ds}{h_{\omega_m^*}[\omega_m^*, \psi(\omega_m^*)]} + \omega_m^*\right] = \frac{h[\omega_m^*, \psi(\omega_m^*)] - k - \int_{\omega_m^0}^{\omega_m^*} h_{\omega_m^*}[s, \psi(s)]ds}{h_{\omega_f^*}[\omega_m^*, \psi(\omega_m^*)]} + \psi(\omega_m^*). \tag{A.11}$$

To complete our proof, we need to establish that $\omega_f^* = \psi(\omega_m^*)$ and that it is strictly monotonically increasing in ω_m^*. Applying the implicit function theorem to (A.11), we get

$$\frac{\partial \psi}{\partial \omega_m^*} = \frac{\gamma'\left[\frac{h_{\omega_m^*}^2[\omega_m^*, \psi(\omega_m^*)] - \left\{k + \int_{\omega_m^0}^{\omega_m^*} h_{\omega_m^*}[s, \psi(s)]ds\right\}h_{\omega_m^*\omega_m^*}(\cdot,\cdot)}{h_{\omega_m^*}^2[\omega_m^*, \psi(\omega_m^*)]} + 1\right] + \left[\frac{c_f^2 h_{\omega_f^*\omega_m^*}(\cdot,\cdot)}{h_{\omega_f^*}^2[\omega_m^*, \psi(\omega_m^*)]}\right]}{\gamma'\left[\frac{c_m^2 h_{\omega_m^*\omega_f^*}(\cdot,\cdot)}{h_{\omega_m^*}^2[\omega_m^*, \psi(\omega_m^*)]}\right] + 1 + \left[\frac{h_{\omega_f^*}^2[\omega_m^*, \psi(\omega_m^*)] - \left\{k' + \int_{\omega_f^0}^{\omega_f^*} h_{\omega_f^*}[s, \psi(s)]ds\right\}h_{\omega_f^*\omega_f^*}(\cdot,\cdot)}{h_{\omega_f^*}^2[\omega_m^*, \psi(\omega_m^*)]}\right]}. \tag{A.12}$$

Recall that, $\forall \omega_m^*, \omega_f^* > 0, h_{\omega_m^*}(\omega_m^*, \omega_f^*), h_{\omega_f^*}(\omega_m^*, \omega_f^*) > 0, h_{\omega_m^* \omega_m^*}(\omega_m^*, \omega_f^*),\ \ h_{\omega_f^* \omega_f^*}(\omega_m^*, \omega_f^*) < 0$, and $h_{\omega_m^* \omega_f^*}(\omega_m^*, \omega_f^*) > 0$. Finally, note that $\gamma'(\cdot) > 0$ due to positive assortative matching. Hence, we establish that the numerator and the denominator of (A.12) is strictly positive. As a result, $\exists \omega_f^* = \psi(\omega_m^*) s.t. \partial\psi/\partial\omega_m^* >$ and equation (A.12) is satisfied. ‖

Proof of Lemma 4. According to Definition 4, the marital production technology $h(\omega_m^*, \omega_f^*)$ is submodular. As a result, if we take two men with the investment levels of ω_m and ω_m' such that $\omega_m' > \omega_m$ and two women with ω_f and ω_f' where $\omega_f' > \omega_f$, then by definition it has to be the case that

$$h(\omega_m', \omega_f') + h(\omega_m, \omega_f) < h(\omega_m', \omega_f) + h(\omega_m, \omega_f'). \tag{A.13}$$

Equation (A.13) implies that

$$h(\omega_m', \omega_f') - h(\omega_m', \omega_f) < h(\omega_m, \omega_f') - h(\omega_m, \omega_f). \tag{A.14}$$

Given (A.14), the man with the investment level ω_m gains more from marrying the woman with the investment level ω_f' than the man with ω_m'. And since utility is fully transferable between spouses, the man with ω_m can outbid the man with ω_m' in the competition to marry the woman with ω_f'. As a result, the pairings (ω_m, ω_f') and (ω_m', ω_f) are maritally sustainable, while the pairings (ω_m', ω_f') and (ω_m, ω_f) are not. ‖

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